

Example 9
Z-test – One Sample
Comparing sample and population means (one-tailed)
Setup

A researcher wants to know if elevation influences people's blood hemoglobin content. From what we know about biology we would expect that people from higher elevations would have more hemoglobin in their blood to compensate for lower oxygen concentrations. The researcher has collected blood samples from 64 adult males that live at high elevations and measured their blood hemoglobin content (data provided below g/100 ml). He wants to know if the sample mean differs significantly from the worldwide population mean of 15.80 g/100 ml with a population standard deviation of 2.00 g/100 ml.

Blood hemoglobin content (g/100ml)

| | | | |
|-------|-------|-------|-------|
| 16.55 | 19.68 | 12.29 | 16.22 |
| 14.13 | 15.56 | 15.28 | 16.42 |
| 15.93 | 14.85 | 17.40 | 16.54 |
| 14.45 | 18.16 | 18.03 | 19.43 |
| 17.83 | 15.50 | 18.22 | 15.16 |
| 14.11 | 14.60 | 16.66 | 16.38 |
| 16.87 | 10.97 | 17.83 | 14.77 |
| 16.98 | 17.91 | 14.97 | 17.28 |
| 17.77 | 17.98 | 16.43 | 15.64 |
| 20.10 | 18.30 | 19.84 | 16.83 |
| 15.78 | 19.00 | 14.58 | 20.02 |
| 15.83 | 16.07 | 17.62 | 16.19 |
| 22.34 | 15.95 | 19.24 | 14.65 |
| 19.82 | 15.90 | 17.09 | 16.90 |
| 19.42 | 18.01 | 11.76 | 15.40 |
| 18.04 | 21.89 | 16.49 | 18.15 |

Example 9

Z-test – One Sample Comparing sample and population means (one-tailed) Solution

Problem: We want to know if people from higher elevations have greater blood hemoglobin content than people in general.

1. State your question: Do people that live at higher elevations have more hemoglobin in their blood?
 - a. Is this a good scientific question? Definable, measurable, and controllable
 - b. Identify your population: Hemoglobin content of people
 - c. Identify your dependent variable: Hemoglobin content of blood
 - d. Identify your independent variable: Elevation
2. State your hypothesis set
 - a. Verbal hypothesis: People that live at high elevation have more hemoglobin content in their blood than people in general.
 - b. Statistical hypothesis (H_0 , H_A):
 - H_0 : $x = \mu$, $x=15.80$ Elevation does not influence blood hemoglobin content of people.
 - H_A : $x > \mu$, $x>15.80$ People that live at high elevation have more hemoglobin content in their blood than people in general.
 - c. Is your hypothesis set exhaustive? No, Truncated
 - d. Is your hypothesis set exclusive? Yes
3. State your significance level: $\alpha = 0.05$
4. Select the appropriate test.
 - a. Variable scale:
 - i. Dependent variable: Ratio
 - o Converted or transformed? No
 - ii. Independent variable: Nominal
 - o Converted or transformed? No
 - b. What information is given or available
 - i. Sample data: Sample mean
 - ii. Parameters: Population mean and Population standard deviation
 - c. Number of samples: 1
 - d. Are the data paired or unpaired? Not Applicable
 - e. What aspect of the variable do you want to compare? Central tendency -- Means
 - f. State the test to be used: Z-Test – One Sample
 - i. Are assumptions met? Yes
 - o Random sample
 - o Independent samples
 - o Normally distributed data – Tested

Shapiro-Wilk normality test

data: HighElev\$Hemoglobin
W = 0.9824, p-value = 0.494

Since $p > 0.05$, the data seems to be normally distributed.

5. Conduct your sampling
We obtained hemoglobin content from blood samples from 64 adult males from high elevation sites.
6. Graph the data

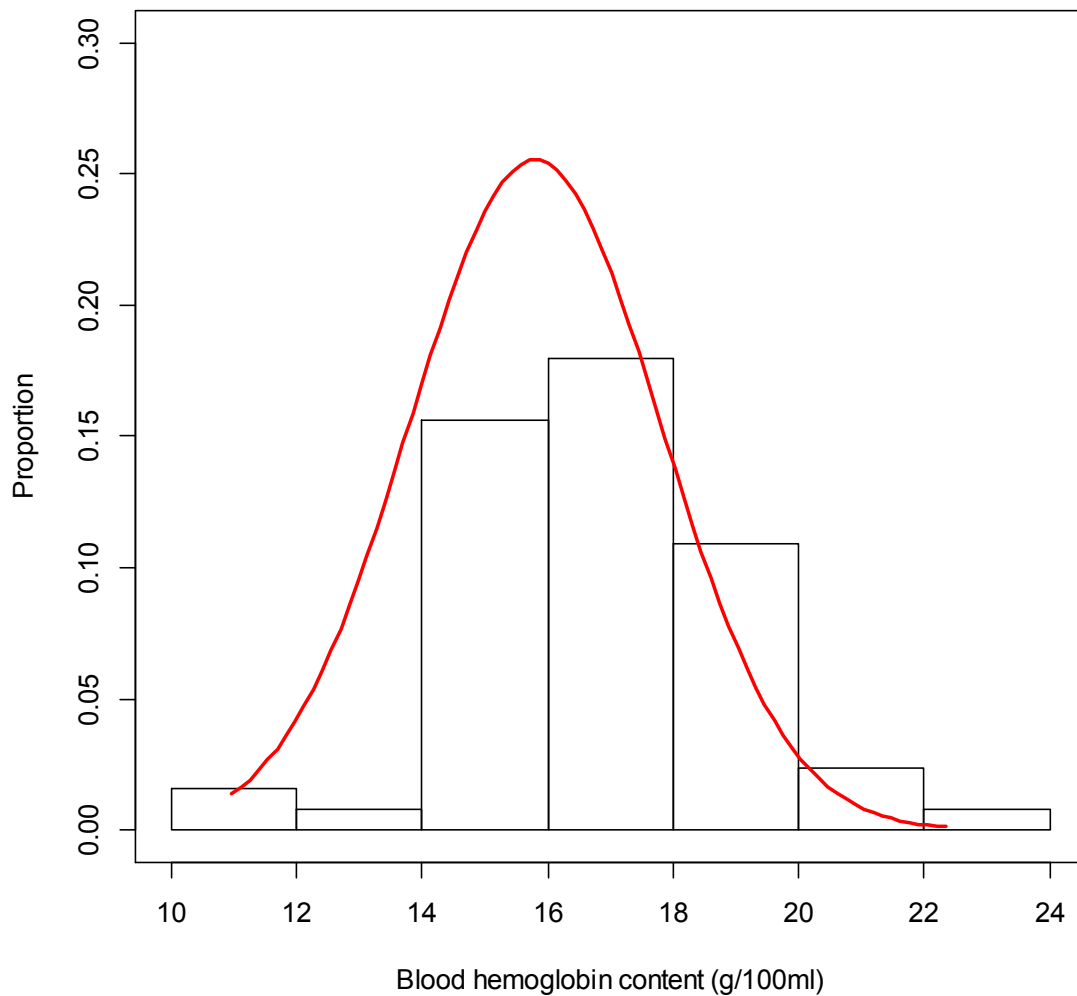


Figure 1. Blood hemoglobin content from 64 men that lived at high elevations. The red line represents the expected normal distribution ($\mu=15.80$, $\sigma=2.00$).

7. Summarize the data

Population parameters (Obtained from many years of research)

$$\mu = 15.80 \text{ g/100 ml}$$

$$\sigma = 2.00 \text{ g/100 ml}$$

Sample description

$$n = 64$$

$$\bar{x} = 16.81 \text{ grams/100 ml}$$

Essentially we are asking is 16.81 grams/100 ml significantly greater than 15.80 grams/100 ml?

8. Calculate your test statistic

One Sample z-test

data: SampleData

$z = 4.0494$, $n = 64.00$, Std. Dev. = 2.00, Std. Dev. of the sample mean = 0.25, $p\text{-value} = 2.568e-05$

alternative hypothesis: true mean is greater than 15.8

95 percent confidence interval:

16.40113 Inf

sample estimates:

mean of SampleData

16.81234

9. Retain or reject your null hypothesis based on your test statistic

The calculated $p\text{-value}$ ($2.568e-05$) is less than the significance level (0.05) **AND** the sample mean (16.81 grams/100 ml) is greater than the population mean (15.80 g/100 ml), therefore we would reject our null hypothesis and retain our alternate hypothesis.

10. Interpret the results in biological terms.

People at high elevations do have greater hemoglobin content than people in general ($Z=4.05$, $n=64$, $p<0.001$).