Example 5
Confidence intervals
Population standard deviation known

Does the new substance added to the diet of infants influence their weight gain?

The population mean is not known so it has to be estimated.

32 infants were given the substance (weight gains in grams)

14.2	295.7
15.0	400.1
22.4	412.5
25.2	422.0
53.4	423.4
104.5	451.3
119.9	455.0
152.2	463.5
153.6	477.0
154.0	481.0
197.5	500.4
233.0	509.0
234.8	521.2
236.8	532.8
283.4	591.0
284.0	760.9

x=311.89g

The sample mean is a point estimate of the population mean

It would be better to have an interval estimate.

$$\mu = \bar{x} \pm Z\sigma_{\bar{x}}$$

The weight gains of infants have been well studied for many years and we can assume that the population standard deviation is 180 g.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{180}{\sqrt{32}} = 31.8g$$

Assume that we want to be 95% (0.95) certain that the population mean lies within the interval, a 95% confidence interval.

Table A

i. 100% - 95% = 5% (0.05)
ii. Because table A only works on half of the distribution: 0.05/2 = 0.025 (two-tailed -- more on this later)
iii. Because table A works on the portion of the distribution between the mean and another value: 0.50 - 0.025 = 0.475
iv. Find the Z-score (standard deviation units) equivalent to 0.4750 --- 1.96.
v. 2.5% (0.025) of the standard normal curve is greater than 1.96, 2.5% (0.025) of the standard normal curve is less than –1.96. 2.5% + 2.5% = 5% hence the 95% confidence interval.

$$\mu = 311.89 \pm 1.96(31.8)$$

Lower limit = 311.89-(1.96)*(31.8)=249.5 g
Upper limit = 311.89+(1.96)*(31.8)=374.2 g

The 95% confidence interval is 250-374 g