# Probabilities

LECTURE 5

## Objectives

- Define terms.
- Calculate simple probabilities.
  - Discrete probabilities
  - Continuous probabilities
- Explain independent events.
- Understand the basics of Boolean logic.

# Chance

- Chance behavior is unpredictable in the short run, but has regular and predictable patterns in the long run.
  - Example: Coin toss
    - A single toss cannot be predicted
    - Many repetitions – a pattern emerges
  - Example: Simulation

# Probability

- The outcome of a random phenomenon is the proportion of the time the outcome would occur in a very long series of repetitions.
  - You can never observe a probability exactly.
    - You could always conduct more repetitions.

## Counting possible outcomes

- Imagine a phenomenon that can occur in any of k different ways, but in only one of those ways at a time.
  - "Flipping a coin" -- Two possible outcomes (heads or tails), but only one is actually observed each time.
  - "Rolling a die" – Six possible outcomes (1, 2, 3, 4, 5, or 6).
  - "Drawing a card" - 52 different outcomes of drawing one card out of a standard deck.
- The probability of any one outcome is 1/(number of possible outcomes).

## Counting possible outcomes

- Now imagine two phenomena. The first phenomenon can occur in $k_1$ different ways. The second phenomenon can occur in $k_2$ different ways.
  - The number of possible outcomes for these two phenomena is $k_1 \text{ X } k_2$
  - The probability of any one combination of outcomes for the phenomena is $1/(k_1 \text{ X } k_2)$

# Counting possible outcomes

- This method of calculating the number of possible outcomes can be extended to any number of phenomena.

# Table 1. Fox litter size distribution

| Litter size | Frequency |
|---|---|
| 3 | 15 |
| 4 | 42 |
| 5 | 34 |
| 6 | 6 |
| 7 | 2 |
| 8 | 1 |

# Example

- What is the probability that I will select a litter size of 7 at random from the previous distribution?
- Number of occurrences of 7 = 2
- Total number of outcomes = 100
- Probability of selecting a 7 = 2/100 = 0.02

# Example

- What is the probability that I will select a litter size of 4 at random from the previous distribution?

# Table 1. Fox litter size distribution

| Litter size | Frequency |
|---|---|
| 3 | 15 |
| 4 | 42 |
| 5 | 34 |
| 6 | 6 |
| 7 | 2 |
| 8 | 1 |

# Example

- What is the probability that I will select a litter size of 4 at random from the previous distribution?
- Number of occurrences of 4 = 42
- Total number of outcomes = 100
- Probability of selecting a 4 = 42/100 = 0.42

# Example

- What is the probability that I will select a litter size ≥5 at random from the previous distribution?

# Table 1. Fox litter size distribution

| Litter size | Frequency |
|---|---|
| 3 | 15 |
| 4 | 42 |
| 5 | 34 |
| 6 | 6 |
| 7 | 2 |
| 8 | 1 |

## Example

- What is the probability that I will select a litter size ≥5 at random from the previous distribution?
- Number of occurrences ≥ 5 = 43
- Total number of outcomes = 100
- Probability of selecting ≥ 5 = 43/100 = 0.43

## Hint

- The trick to calculating probabilities is
  - counting the number of possible "successes. "
  - counting the number of possible outcomes.

# Probability models

- The description of an event has 2 parts.
  - A list of possible outcomes.
  - A probability for each outcome.

  - Sample space S of a random phenomenon is the full set of all possible outcomes.
  - An event is an outcome or a set of outcomes of a random phenomenon. An event is a subset (sample) of sample space S.
  - Probability model is a mathematical description of a random phenomenon consisting of two parts – sample space S and a way of estimating probabilities.

# Probability rules

- A probability is a number between 0 and 1.
  - Proportion

P(A) – probability of event A

$0 < P(A) \leq 1$

# Probability rules

- All possible outcomes together must have a probability of 1.

  S – sample space

  P(S) = 1

# Probability rules

- If two events have no outcomes in common (disjoint), the probability that one or the other outcome occurs is the sum of the their individual probablities.

  A and B are disjoint events

  p(A or B) = P(A) + P(B)

  more on this in a moment

# Probability rules

- The probability that an event does not occur is 1 minus the probability that the event does occur.

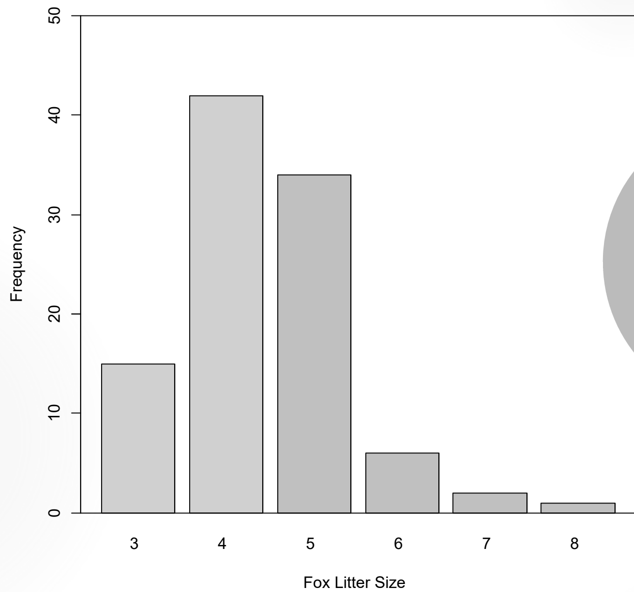
  P(not A) = 1 - P(A)

# Discrete probabilities

- The outcomes are discrete outcomes
  - Only specific outcomes are possible
  - Discrete variables

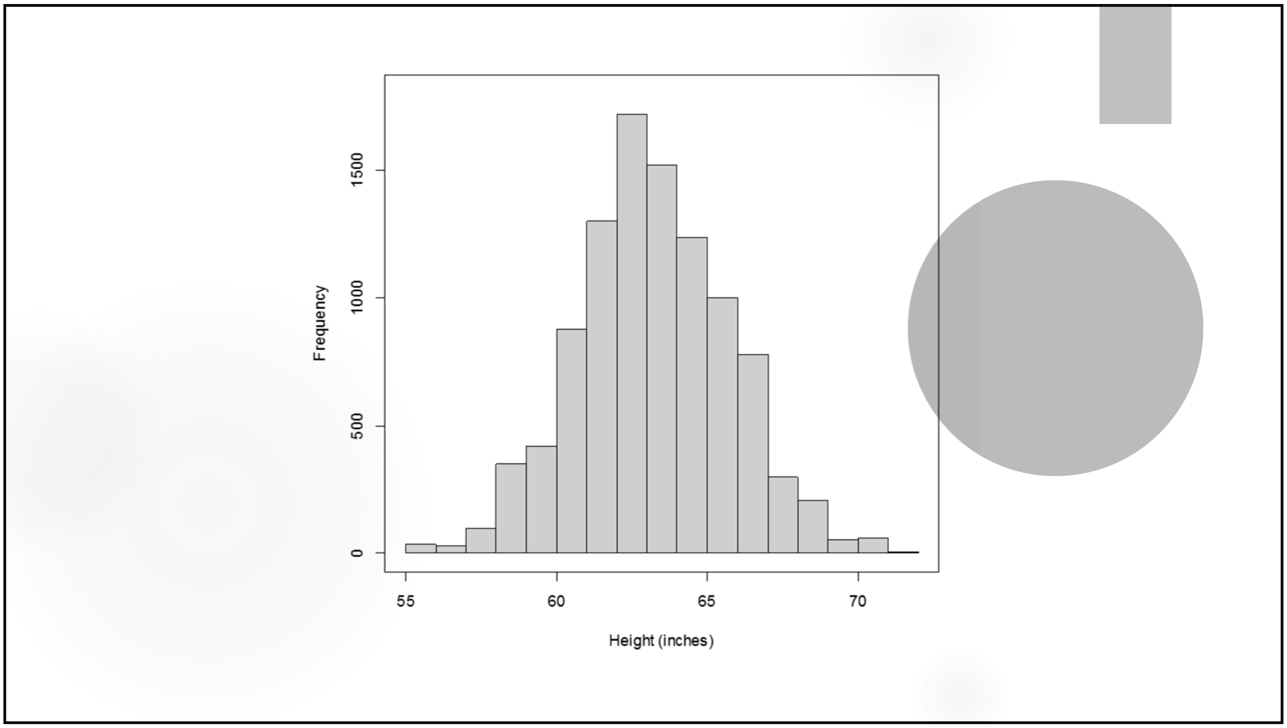
  - Our example of fox litter size represent discrete probabilities.

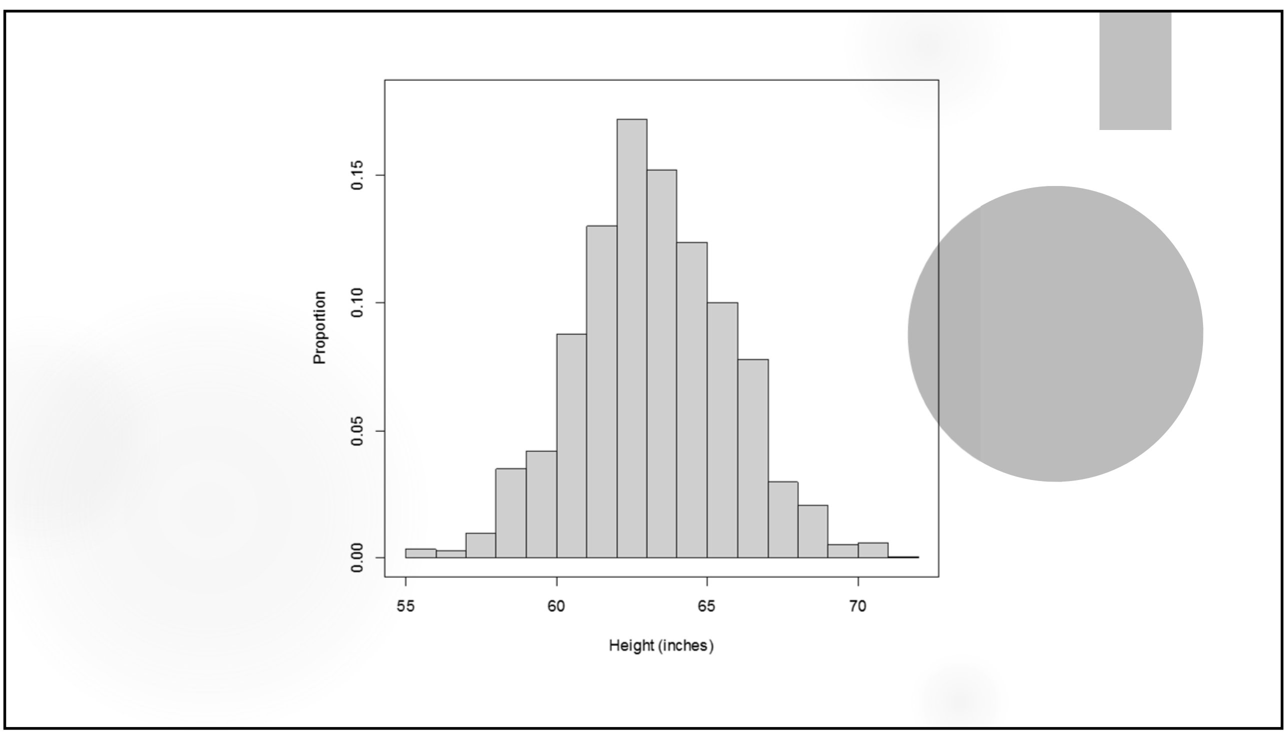
    - Let's redo the last example problem and instead of using a table visualize the problem as a graph.




# Continuous probabilities

- The outcomes are continuous outcomes
- Continuous variables
  - Histograms instead of bar graphs
  - Imagine the heights of 10,000 adult American women.


Frequency
0
500
1000
1500
55
60
65
70
Height (inches)


Proportion
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0.05
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0.15
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60
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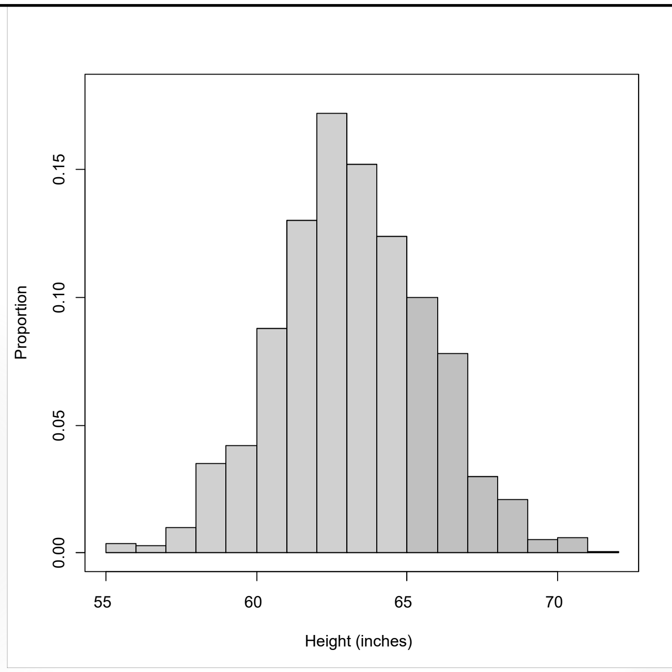
Height (inches)


Proportion
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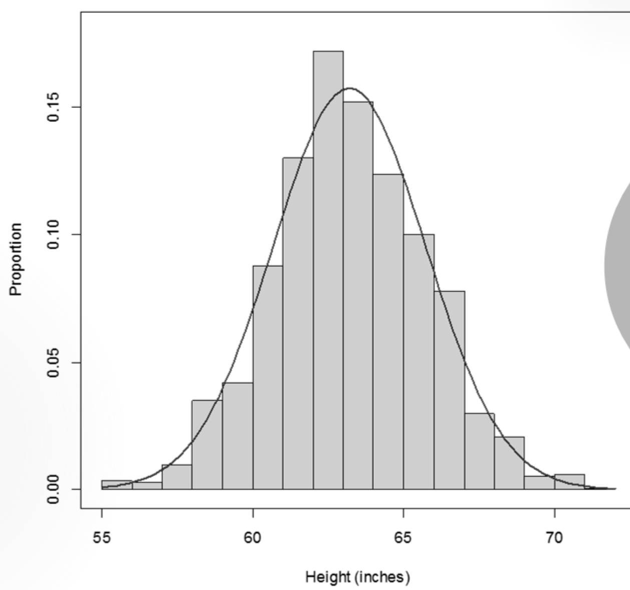
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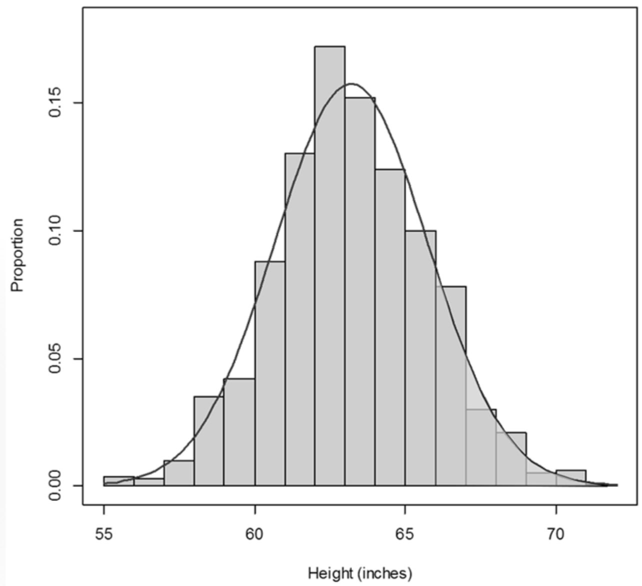


# Continuous probabilities

- The probability of any specific outcome is 0.
  - There is no area associated with a single value so there is no associated probability.
  - Must be a range.

# Probability

- Independent events

# Probability of an event

- The relative frequency of an event is proportion of the total observations of outcomes that event represents.
- Discuss Boolean **and** and **or**
- Discuss Venn diagrams

# Probability of an event

- Adding probabilities
- If outcomes are mutually exclusive then the probability can be calculated as:

P(A **or** B) = P(A) + P(B)

# Probability of an event

- If two or more distinct events can occur at the same time, the probability of an intersection of events is the product of the probabilities of the individual events:

P(A **and** B) = [P(A)][P(B)]